# 2020 Summer JMO (Day II)

*July 27, 2020 to August 24, 2020*

*Note:* For any geometry problem whose statement begins with an asterisk (∗), the first page of the solution must be a large, in-scale, clearly labeled diagram. Failure to meet this requirement will result in an automatic 1-point deduction.

**SJMO 4.** (∗) Let $B$ and $C$ be points on a semicircle with diameter $AD$ such that $B$ is closer to $A$ than $C$. Diagonals $AC$ and $BD$ intersect at point $E$. Let $P$ and $Q$ be points such that $\overline{PE} \perp \overline{BD}$ and $\overline{PB} \perp \overline{AD}$, while $\overline{QE} \perp \overline{AC}$ and $\overline{QC} \perp \overline{AD}$. If $BQ$ and $CP$ intersect at point $T$, prove that $\overline{TE} \perp \overline{BC}$.

**SJMO 5.** A nondegenerate triangle with perimeter 1 has side lengths $a, b$, and $c$. Prove that

$$\left|\frac{a-b}{c+ab}\right| + \left|\frac{b-c}{a+bc}\right| + \left|\frac{c-a}{b+ac}\right| < 2.$$

**SJMO 6.** We say a positive integer $n$ is *$k$-tasty* for some positive integer $k$ if there exists a permutation $(a_0, a_1, a_2, \ldots, a_n)$ of $(0, 1, 2, \ldots, n)$ such that $|a_{i+1} - a_i| \in \{k, k+1\}$ for all $0 \le i \le n-1$. Prove that for all positive integers $k$, there exists a finite $N$ such that all integers $n \ge N$ are $k$-tasty.

*Time: 4 hours and 30 minutes.*
*Each problem is worth 7 points.*